If n is of the form $4i + 2$,

$$s_0 = 2^{n-2},$$

$$s_1 = 2^{n-2} + \nu\, 2^{\frac{n-2}{2}},$$

$$s_2 = 2^{n-2},$$

$$s_3 = 2^{n-2} - \nu\, 2^{\frac{n-2}{2}};$$

If n is of the form $4i + 3$,

$$s_0 = 2^{n-2} - \nu\, 2^{\frac{n-3}{2}},$$

$$s_1 = 2^{n-2} + \nu\, 2^{\frac{n-3}{2}},$$

$$s_2 = 2^{n-2} + \nu\, 2^{\frac{n-3}{2}},$$

$$s_3 = 2^{n-2} - \nu\, 2^{\frac{n-3}{2}}.\text{''}$$

The proof of this rests upon the equations

$$s'_3 - s'_2 = s_3 - s_1,$$

$$s'_2 - s'_1 = s_2 - s_0,$$

$$s'_1 - s'_0 = s_1 - s_3,$$

$$s'_0 - s'_3 = s_0 - s_2,$$

combined with $s_0 + s_1 + s_2 + s_3 + 2^n$.

Though the theorems which I have now stated or indicated are not devoid of interest, I should hardly have brought them under the notice of the Academy if they had not led Sir William R. Hamilton to discuss the more general question treated of in the Note appended to this paper. It is at his suggestion that I have communicated the substance of the letter which I addressed to him on this subject.

I may be allowed to add, that the first theorem stated in this paper was suggested by the investigation of a very simple geometrical problem, and that I have found that it admits of being very curiously illustrated by means of my theory of algebraic triplets.

EXTRACT *from a recent Manuscript Investigation, suggested by a Theorem of* DEAN GRAVES, *which was contained in a Letter received by me a week ago.*

1. Let n_r, for any whole value not less than zero of n, and for *any* whole value of r, be defined to be the (always whole) coefficient of the power

x^2, in the expansion of $(1+x)^n$ for an arbitrary x; so that we have always $n_0 = 1$, but $n_r = 0$ in each of the two cases, $r < 0$, $r > n$.

2. Let p be any whole number > 0; and let the sum of all the coefficients n_m, for which $m \equiv r$ (mod. p), the value of n being given, be denoted by the symbol,

$$s^{(p)}_{n,\,r};$$

which thus represents, when n and p are given, a periodical function of r, in the sense that

$$s^{(p)}_{n,\,r} = s^{(p)}_{n,r\,+\,tp},$$

if t be *any* whole number (positive or negative).

3. A fundamental property of the binomial coefficient n_r is expressed by the equation,

$$(n+1)_r = n_r + n_{r-1};$$

from which follows at once this analogous *equation in differences*,

$$s^{(p)}_{n+1,r} = s^{(p)}_{n,\,r} + s^{(p)}_{n,r-1};$$

with the p *initial values*,

$$s^{(p)}_{0,0} = 1,\ s^{(p)}_{0\ 1} = 0,\ s^{(p)}_{0,\,2} = 0,\ \ldots\ s^{(p)}_{0,\,p-1} = 0.$$

4. Hence may be deduced the *general expression*,

$$s^{(p)}_{n,\,r} = p^{-1}\ \Sigma x^{-r}\,(1+x)^n;$$

in which the summation is to be effected with respect to the p roots x, of the binomial equation,

$$x^p - 1 = 0.$$

5. The summand *term*,

$$x^{-r}(1+x)^n,$$

usually involves *imaginaries*, which must however disappear in the *result*; and thus the general expression for the *partial sum*, s, may be reduced to the *real* and *trigonometrical form*,

$$s^{(p)}_{n,r} = p^{-1}\,\Sigma\left(2\cos\frac{m\pi}{p}\right)^n \cos\frac{m(n-2r)\pi}{p},$$

with the verification that

$$0 = \Sigma\left(2\cos\frac{m\pi}{p}\right)^n \sin\frac{m(n-2r)\pi}{p};$$

each summation being performed with respect to an auxiliary integer m, from $m=0$ to $m=1$.

6. Accordingly, *without* using *imaginaries*, it is easy to prove that this expression (5) satisfies all the recent conditions (3), and is therefore a correct expression for the partial sum

$$s^{(p)}_{n,r};$$

while a similar proof of the recent equation $0 = \&c.$

7. But to form *practically*, with the easiest possible *arithmetic*, a *Table of Values* of s, for any given *period*, p, we are led by No. 3 to construct a *Scheme*, such as the following:—

TABLE OF VALUES OF $s^{(5)}_{n,r}$.

	$r=5$	4	3	2	1	0	Verification.
$n=0$	$s=1$	0	0	0	0	1	$\Sigma s=1$
1	1	0	0	0	1	1	2
2	1	0	0	1	2	1	4
3	1	0	1	3	3	1	8
4	1	1	4	6	4	1	16
5	2	5	10	10	5	2	32
6	7	15	20	15	7	7	64

The PRESIDENT read the following paper by the late Sir WILLIAM R. HAMILTON :—

ON A NEW SYSTEM OF TWO GENERAL EQUATIONS OF CURVATURE,

Including as easy consequences a new form of the Joint Differential Equation of the Two Lines of Curvature, with a new Proof of their General Rectangularity; and also a new Quadratic for the Joint Determination of the Two Radii of Curvature: all deduced by Gauss's Second Method, for discussing generally the Properties of a Surface; and the latter being verified by a Comparison of Expressions, for what is called by him the Measure of Curvature.

1. NOTWITHSTANDING the great beauty and importance of the investigations of the illustrious GAUSS, contained in his *Disquisitiones Generales circà Superficies Curvas*, a Memoir which was communicated to the *Royal Society of Göttingen* in October, 1827, and was printed in Tom. vi. of